Logics of Rational Agency

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- Philosophy (social philosophy, epistemology)
- Game Theory
- Social Choice Theory
- AI (multiagent systems)

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What is a rational agent?

- maximize expected utility (instrumentally rational)
- react to observations
- revise beliefs when learning a *surprising* piece of information
- understand higher-order information
- plans for the future
- asks questions
- ????

We are interested in reasoning about rational agents interacting in *social* situations.

There is a jungle of formal systems!

- logics of informational attitudes (knowledge, beliefs, certainty)
- logics of action & agency
- temporal logics/dynamic logics
- logics of motivational attitudes (preferences, intentions)

(*Not to mention various game-theoretic/social choice models and logical languages for reasoning about them*)

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There is a jungle of formal systems!

- How do we compare different logical systems studying the same phenomena?
- How *complex* is it to reason about rational agents?
- (How) should we *merge* the various logical systems?
- What do the logical frameworks contribute to the discussion on rational agency?

and logical languages for reasoning about them)

We are interested in reasoning about rational agents interacting in *social* situations.

- playing a game (eg. a card game)
- having a conversation
- executing a *social procedure*
- ....

What about *game-theoretic* analyses?

Goal: incorporate/extend existing game-theoretic/social choice analyses

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R. Aumann and J. H. Dreze. *Rational Expectation in Games*. American Economic Review (2008).

Logics of Rational Agency

Basic Ingredients

- What are the basic building blocks? (the nature of time (continuous or discrete/branching or linear), how (primitive) *events* or *actions* are represented, how *causal* relationships are represented and what constitutes a *state of affairs*.)

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- What are the primitive operators?
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- Static vs. dynamic

Basic Ingredients

- informational attitudes
- time, actions and ability
- motivational attitudes

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LP: "P is an epistemic possibility"

KLP: "Ann knows that she thinks P is possible"

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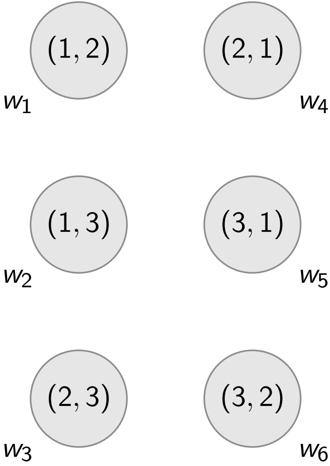

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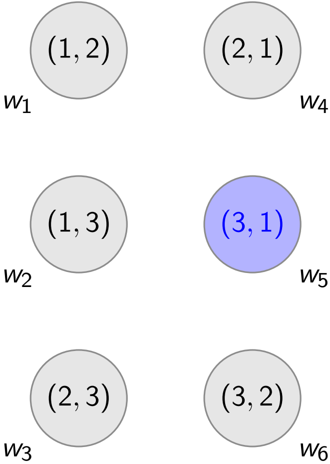

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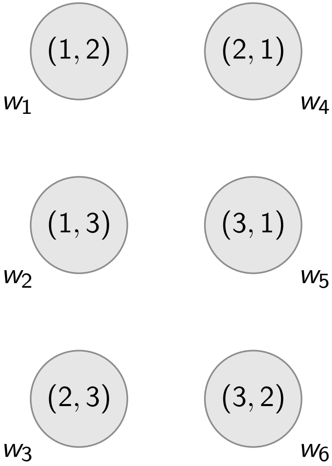

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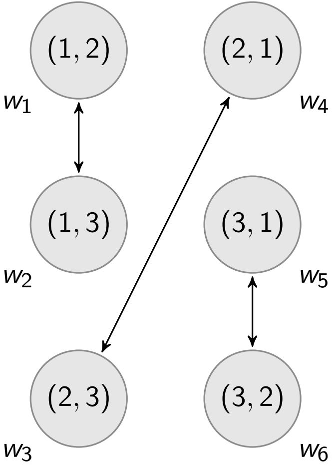

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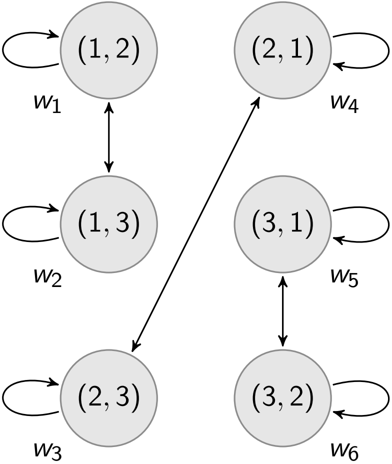

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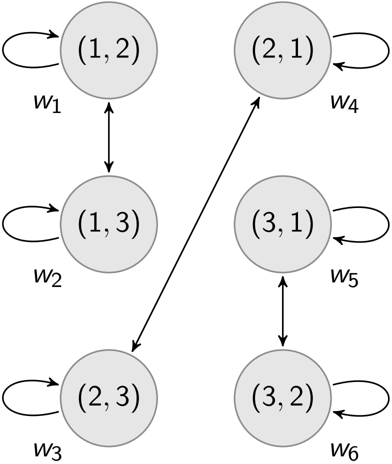
Eg., $V(H_1) = \{w_1, w_2\}$



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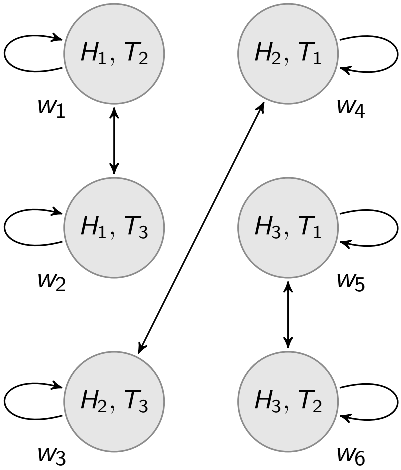

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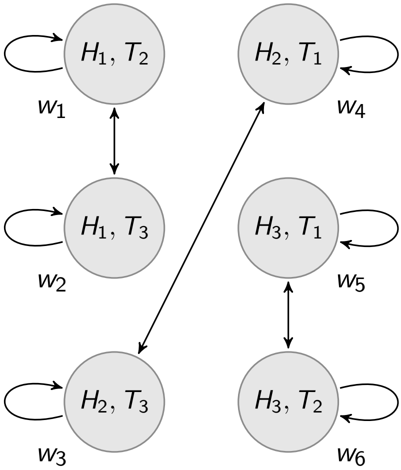

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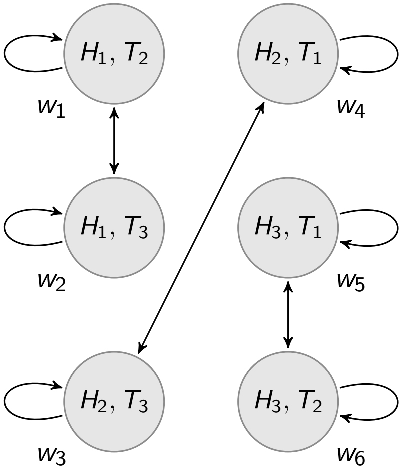

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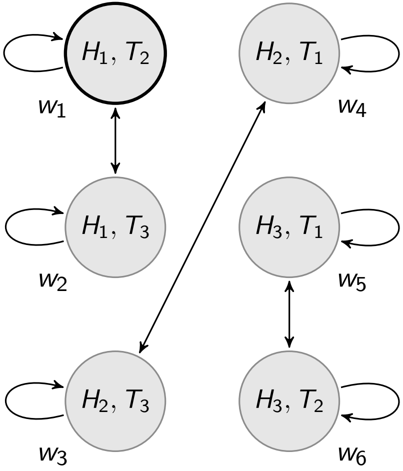

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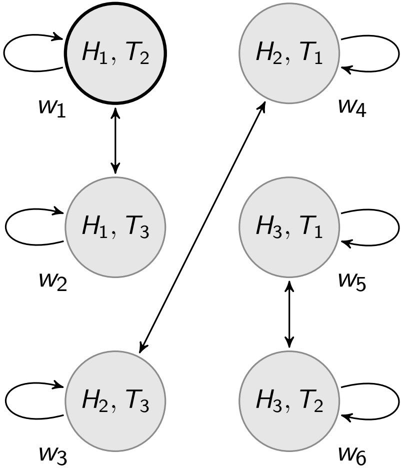

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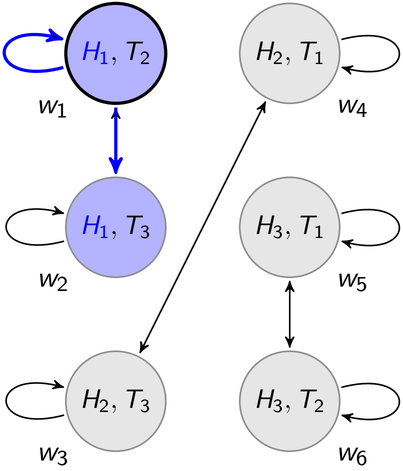

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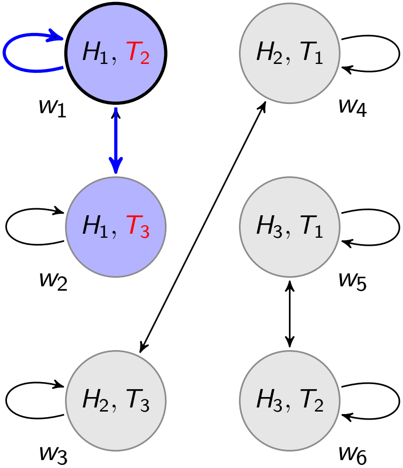

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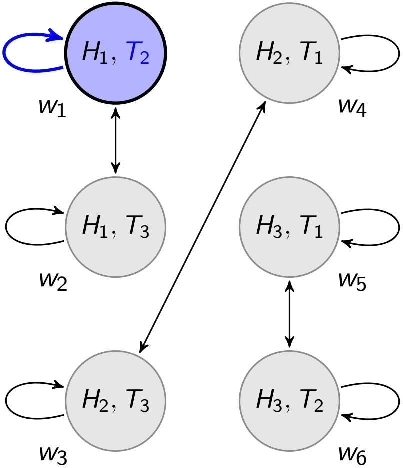

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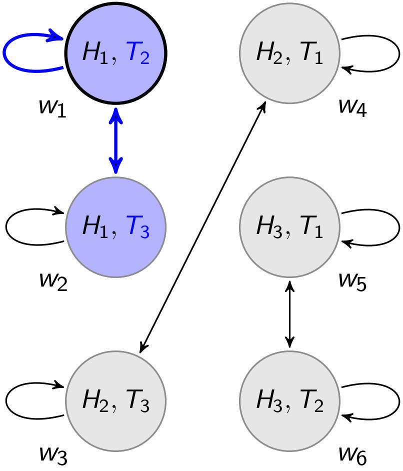
$\mathcal{M}, w_1 \models K(T_2 \vee T_3)$



The Language

: $\varphi \ := \ p \mid \neg\varphi \mid \varphi \wedge \psi \mid K\varphi$

Kripke Models: $\mathcal{M} = \langle W, R, V \rangle$ and $w \in W$

Truth: $\mathcal{M}, w \models \varphi$ is defined as follows:

- $\mathcal{M}, w \models p$ iff $w \in V(p)$ (with $p \in \text{At}$)
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- $\mathcal{M}, w \models K\varphi$ if for each $v \in W$, if wRv, then $\mathcal{M}, v \models \varphi$

Some Questions

Should we make additional assumptions about R (i.e., reflexive, transitive, etc.)

What idealizations have we made?

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$\neg K\bot$	Serial	Consistency

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- $K_A K_B \varphi$: "Ann knows that Bob knows φ"
- $K_A(K_B\varphi \vee K_B\neg\varphi)$: "Ann knows that Bob knows whether φ
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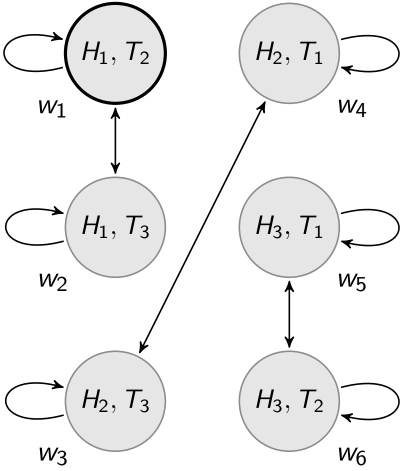

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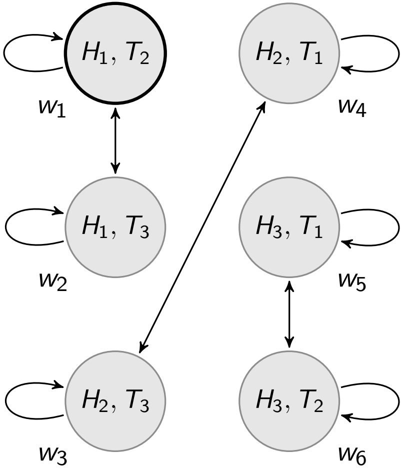

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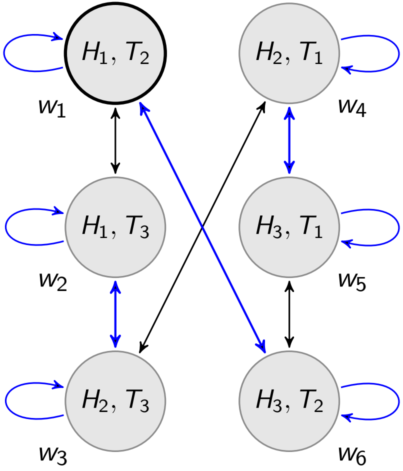

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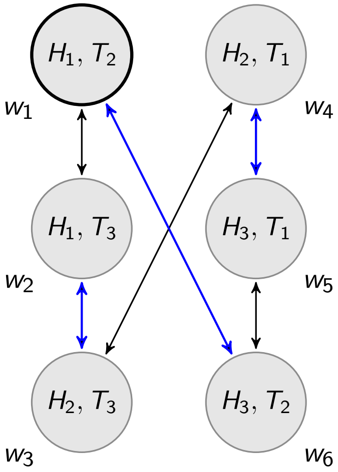

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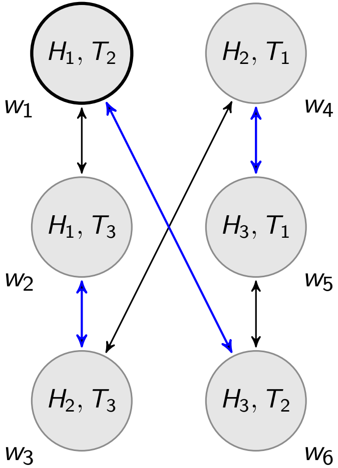

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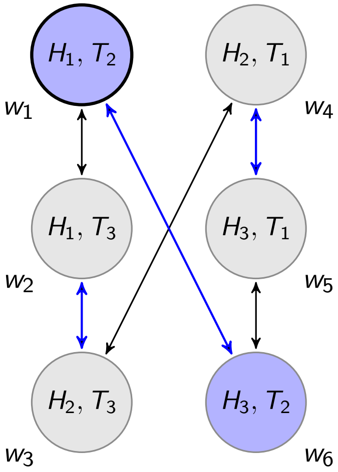

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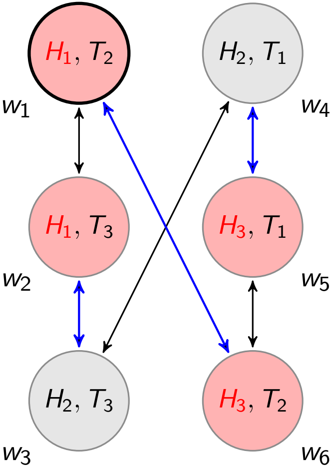

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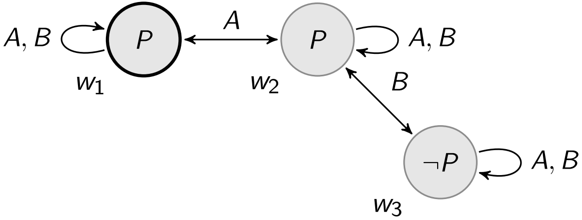

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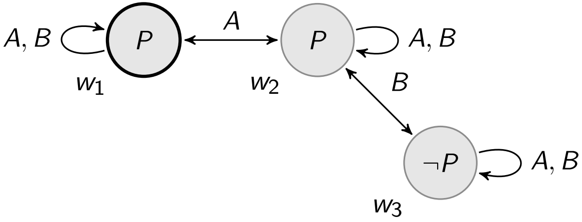

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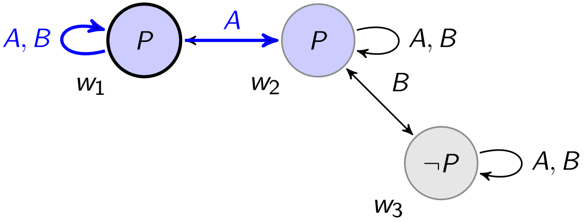

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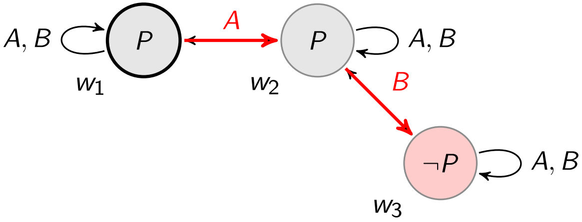
Common Knowledge

CP: "It is common knowledge that P" — "Everyone knows that everyone knows that everyone knows that $\cdots$ P".

Is *common knowledge* different from *everyone knows*?



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Common Knowledge

The operator "everyone knows P", denoted EP, is defined as follows

$$EP \quad := \quad \bigwedge_{i \in \mathcal{A}} K_i P$$

$w \models CP$ iff every finite path starting at w ends with a state satisfying P.

$$CP \longrightarrow ECP$$

$$CP \rightarrow ECP$$

Suppose you are told "Ann and Bob are going together,"' and respond "sure, that's common knowledge." What you mean is not only that everyone knows this, but also that the announcement is pointless, occasions no surprise, reveals nothing new; in effect, that the situation after the announcement does not differ from that before. ...the event "Ann and Bob are going together" — call it P — is common knowledge if and only if some event — call it Q — happened that entails P and also entails all players' knowing Q (like all players met Ann and Bob at an intimate party). *(Robert Aumann)*

$$P \wedge C(P \rightarrow EP) \rightarrow CP$$

An Example

Two players Ann and Bob are told that the following will happen. Some positive integer n will be chosen and *one* of n, $n+1$ will be written on Ann's forehead, the other on Bob's. Each will be able to see the other's forehead, but not his/her own.

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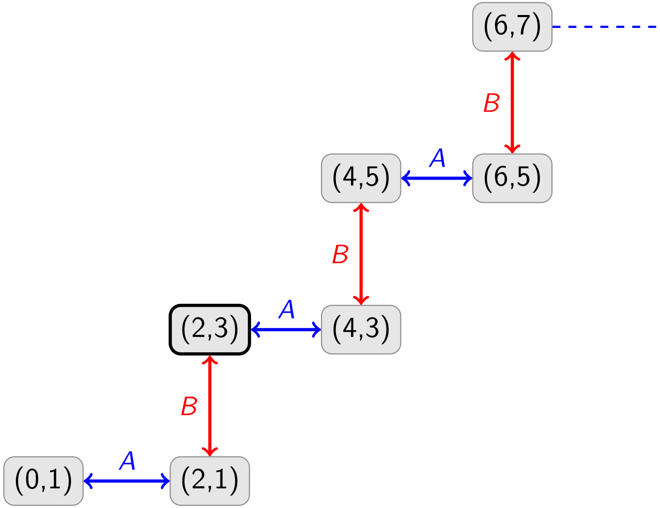
(6,7)
B
(4,5)
A
(6,5)
B
(2,3)
A
(4,3)
B
(0,1)
A
(2,1)

Basic Ingredients

- ✓ informational attitudes (knowledge, group knowledge, belief, certainty, etc.)
- time, actions and ability
- motivational attitudes

Actions: Two Views

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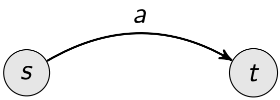
1. Actions *transition between states, or situations*



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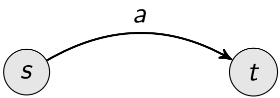
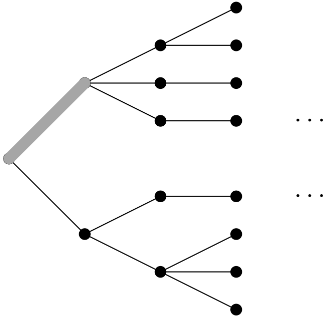


2. Actions restrict the set of *possible future histories*



Propositional Dynamic Logic

Semantics: $\mathcal{M} = \langle W, \{R_a \mid a \in \mathrm{P}\}, V\rangle$ where for each $a \in \mathrm{P}$, $R_a \subseteq W \times W$ and $V : \mathrm{At} \to \wp(W)$

- $R_{\alpha\cup\beta} := R_\alpha \cup R_\beta$
- $R_{\alpha;\beta} := R_\alpha \circ R_\beta$
- $R_{\alpha^*} := \cup_{n\geq 0} R_\alpha^n$
- $R_{\varphi?} = \{(w, w) \mid \mathcal{M}, w \models \varphi\}$

$$\mathcal{M}, w \models [\alpha]\varphi \text{ iff for each } v\text{, if } wR_\alpha v \text{ then } \mathcal{M}, v \models \varphi$$

Background: Propositional Dynamic Logic

1. Axioms of propositional logic
2. $[\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\alpha]\psi)$
3. $[\alpha \cup \beta]\varphi \leftrightarrow [\alpha]\varphi \wedge [\beta]\varphi$
4. $[\alpha; \beta]\varphi \leftrightarrow [\alpha][\beta]\varphi$
5. $[\psi?]\varphi \leftrightarrow (\psi \rightarrow \varphi)$
6. $\varphi \wedge [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi$
7. $\varphi \wedge [\alpha^*](\varphi \rightarrow [\alpha]\varphi) \rightarrow [\alpha^*]\varphi$
8. Modus Ponens and Necessitation (for each program α)

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6. $\varphi \wedge [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi$ (Fixed-Point Axiom)
7. $\varphi \wedge [\alpha^*](\varphi \rightarrow [\alpha]\varphi) \rightarrow [\alpha^*]\varphi$ (Induction Axiom)
8. Modus Ponens and Necessitation (for each program α)

Propositional Dynamic Logic

Theorem PDL is sound and weakly complete with respect to the Segerberg Axioms.

Theorem The satisfiability problem for **PDL** is decidable (EXPTIME-Complete).

D. Kozen and R. Parikh. *A Completeness proof for Propositional Dynamic Logic*. .

D. Harel, D. Kozen and Tiuryn. *Dynamic Logic*. 2001.

Actions and Ability

An early approach to interpret PDL as logic of actions was put forward by Krister Segerberg.

Segerberg adds an "agency" program to the PDL language δA where A is a formula.

K. Segerberg. *Bringing it about*. JPL, 1989.

Actions and Agency

The intended meaning of the program 'δA' is that the agent "brings it about that A': *formally*, δA is the set of all paths p such that

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The axioms:

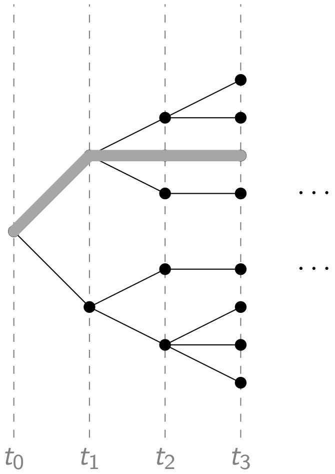
1. $[\delta A]A$
2. $[\delta A]B \rightarrow ([\delta B]C \rightarrow [\delta A]C)$

Actions and Agency

J. Horty. *Agency and Deontic Logic*. 2001.

Logics of Action and Agency

Alternative accounts of agency do not include explicit description of the actions:



STIT

- Each node represents a choice point for the agent.
- A **history** is a maximal branch in the above tree.
- Formulas are interpreted at history moment pairs.
- At each moment there is a choice available to the agent (partition of the histories through that moment)
- The key modality is $[stit]\varphi$ which is intended to mean that the agent i can "see to it that φ is true".
  - $[stit]\varphi$ is true at a history moment pair provided the agent can choose a (set of) branch(es) such that every future history-moment pair satisfies φ

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We use the modality '$\Diamond$' to mean historic possibility.

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$\Diamond[\mathit{stit}]\varphi$: "the agent has the ability to bring about φ".

Example Consider the example of an agent (call her Ann) throwing a dart. Suppose Ann is not a very good dart player, but she just happens to throw a bull's eye. Intuitively, we do not want to say that Ann has the *ability* to throw a bull's eye even though it happens to be true. That is, the following principle should be falsifiable:

$$\varphi \rightarrow \Diamond[\mathit{stit}]\varphi$$

STIT

Example Continuing with this example, suppose that Ann has the ability to hit the dart board, but has no other control over the placement of the dart. Now, when she throws the dart, as a matter of fact, it will either hit the top half of the board or the bottom half of the board. Since, Ann has the ability to hit the dart board, she has the ability to either hit the top half of the board or the bottom half of the board.

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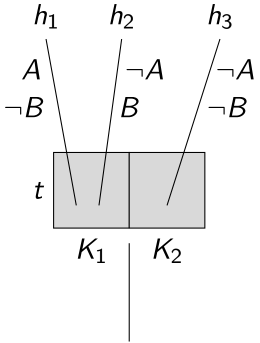
However, intuitively, it seems true that Ann does not have the ability to hit the top half of the dart board, and also she does not have the ability to hit the bottom half of the dart board. Thus, the following principle should be falsifiable:

$$\Diamond[\mathit{stit}](\varphi \vee \psi) \rightarrow \Diamond[\mathit{stit}]\varphi \vee \Diamond[\mathit{stit}]\psi$$

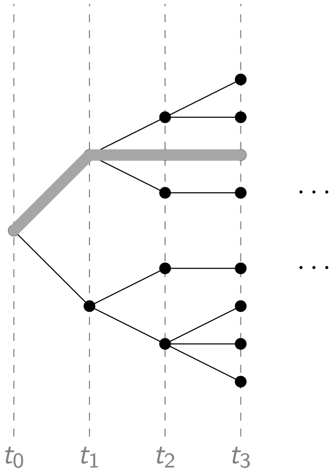
STIT

The following model will falsify both of the above formulas:

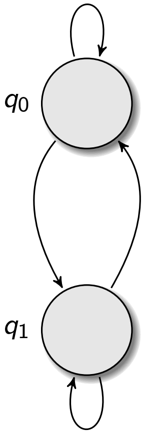


J. Horty. *Agency and Deontic Logic*. 2001.

Temporal Logics



Computational vs. Behavioral Structures



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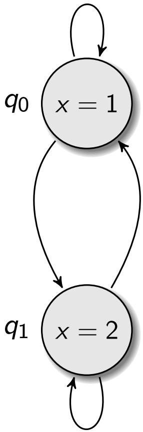

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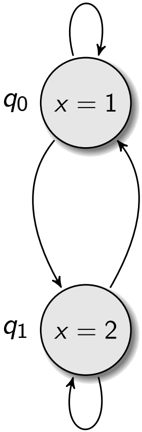

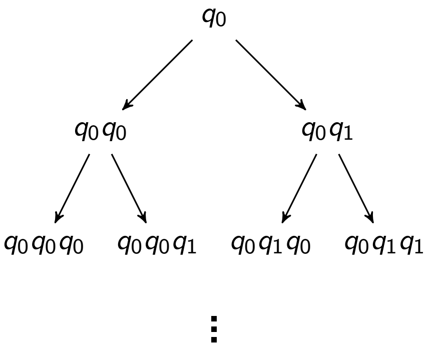

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- *Linear Time Temporal Logic*: Reasoning about computation paths:

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A. Pnuelli. *A Temporal Logic of Programs*. in *Proc. 18th IEEE Symposium on Foundations of Computer Science* (1977).

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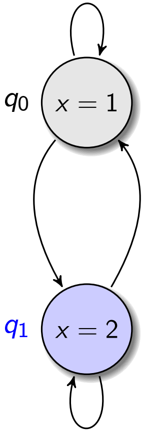
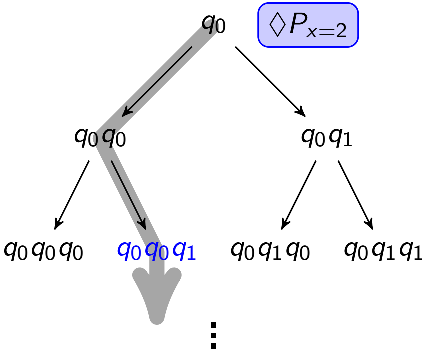
- *Branching Time Temporal Logic*: Allows quantification over paths:

  $\exists\Diamond\varphi$: there is a path in which φ is eventually true.

E. M. Clarke and E. A. Emerson. *Design and Synthesis of Synchronization Skeletons using Branching-time Temproal-logic Specifications*. In *Proceedings Workshop on Logic of Programs*, LNCS (1981).

Temporal Logics





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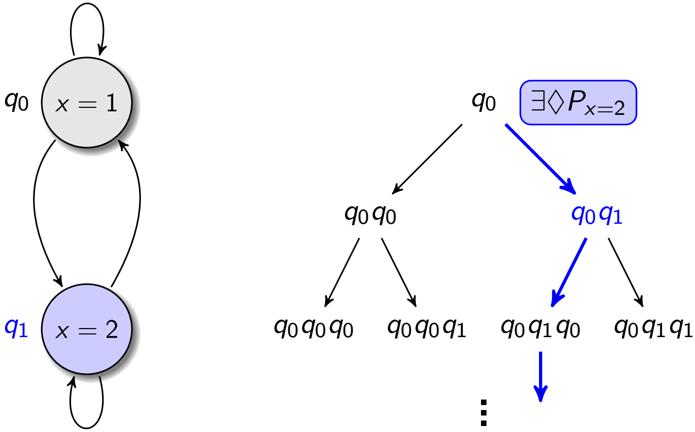

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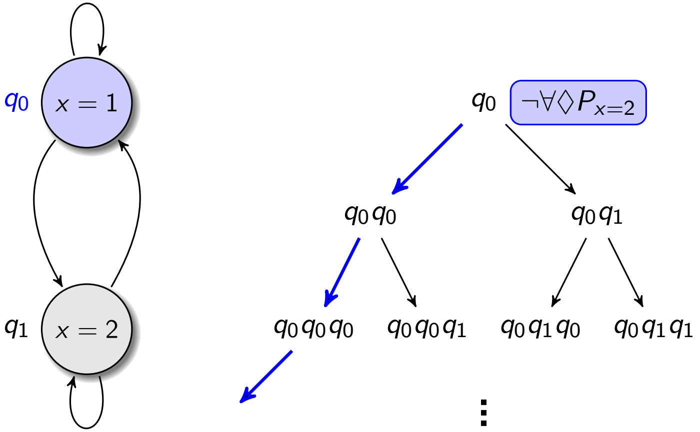

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From Temporal Logic to Strategy Logic

- *Coalitional Logic*: Reasoning about (local) group power.

  $[C]\varphi$: coalition C has a **joint action** to bring about φ.

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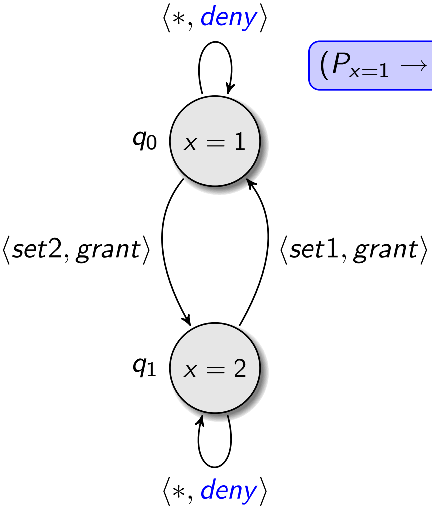
- *Alternating-time Temporal Logic*: Reasoning about (local and global) group power:

  $\langle\langle A\rangle\rangle\Box\varphi$: The coalition A has a **joint action** to ensure that φ will remain true.

R. Alur, T. Henzinger and O. Kupferman. *Alternating-time Temproal Logic*. *Jouranl of the ACM* (2002).

Multi-agent Transition Systems



$$(P_{x=1} \rightarrow [s]P_{x=1}) \wedge (P_{x=2} \rightarrow [s]P_{x=2})$$

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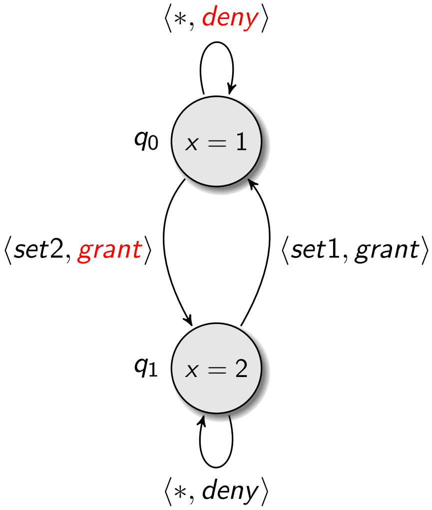

$P_{x=1} \rightarrow \neg[s]P_{x=2}$

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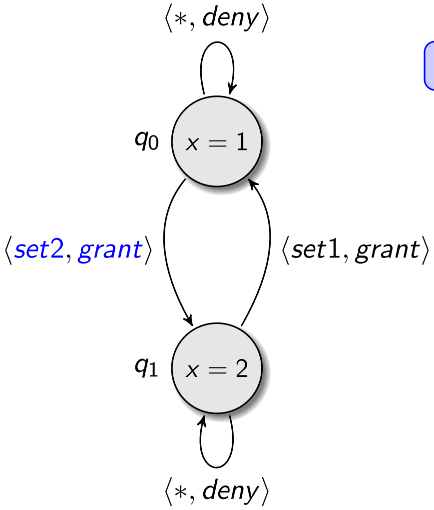

$P_{x=1} \longrightarrow \neg[s, c]P_{x=2}$

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- ✓ informational attitudes (knowledge, group knowledge, belief, certainty, etc.)
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Preference (Modal) Logics

x, y objects

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$x \succeq y$: x is at least as good as y

1. $x \succeq y$ and $y \not\succeq x$ ($x \succ y$)
2. $x \not\succeq y$ and $y \succeq x$ ($y \succ x$)
3. $x \succeq y$ and $y \succeq x$ ($x \sim y$)
4. $x \not\succeq y$ and $y \not\succeq x$ ($x \perp y$)

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Properties: transitivity, connectedness, etc.

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Preference (Modal) Logics

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2. $\langle\succeq\rangle\langle\succ\rangle\varphi \rightarrow \langle\succ\rangle\varphi$
3. $\varphi \wedge \langle\succeq\rangle\psi \rightarrow (\langle\succ\rangle\psi \vee \langle\succeq\rangle(\psi \wedge \langle\succeq\rangle\varphi))$
4. $\langle\succ\rangle\langle\succeq\rangle\varphi \rightarrow \langle\succ\rangle\varphi$

Theorem The above logic (with Necessitation and Modus Ponens) is sound and complete with respect to the class of preference models.

J. van Benthem, O. Roy and P. Girard. *Everything else being equal: A modal logic approach to* ceteris paribus *preferences*. JPL, 2008.

Preference Modalities

$\varphi \geq \psi$: the state of affairs φ is at least as good as ψ (ceteris paribus)

G. von Wright. *The logic of preference*. Edinburgh University Press (1963).

From worlds to sets and back

Lifting

- $X \geq_{\forall\exists} Y$ if $\forall y \in Y\ \exists x \in X$: $x \succeq y$

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Deriving

$P_1 \gg P_2 \gg P_3 \gg \cdots \gg P_n$

$x > y$ iff x and y differ in at least one P_i and the first P_i where this happens is one with $P_i x$ and $\neg P_i y$

F. Liu and D. De Jongh. *Optimality, belief and preference*. 2006.

The Logic of Group Decisions

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Fundamental Problem: groups are inconsistent!

The Logic of Group Decisions: The Doctrinal "Paradox" (Kornhauser and Sager 1993)

p: a valid contract was in place

q: there was a breach of contract

r: the court is required to find the defendant liable.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions: The Doctrinal “Paradox” (Kornhauser and Sager 1993)

Should we accept r?

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

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Should we accept r? No, a simple majority votes no.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions: The Doctrinal "Paradox" (Kornhauser and Sager 1993)

Should we accept r? Yes, a majority votes yes for p and q and $(p \wedge q) \leftrightarrow r$ is a legal doctrine.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Discursive Dilemma

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1	True	True	

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1	True	True	True

Discursive Dilemma

a: "Carbon dioxide emissions are above the threshold x"

$a \rightarrow b$: "If carbon dioxide emissions are above the threshold x, then there will be global warming"

b "There will be global warming"

	a	$a \rightarrow b$	b
1	True	True	True
2	True	False	

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Majority			

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Conclusion: Groups are inconsistent, difference between 'premise-based' and 'conclusion-based' decision making, ...

Group Preference Logics

H. Andréka, M. Ryan and P Yves Schobbens. *Operators and laws for combining preference relations*. Journal of Logic and Computation, 2002.

P. Girard. *Modal Logic for Lexicographic Preference Aggregation*. Manuscript, 2008.

Basic Ingredients

- ✓ informational attitudes (knowledge, group knowledge, belief, certainty, etc.)
- ✓ time, actions and ability (individual and coalitional ability)
- ✓ motivational attitudes (individual preferences, group preferences)

General Issues

Once a semantics and language are fixed, then standard questions can be asked: eg. develop a proof theory, completeness, decidability, model checking.

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- Comparing different frameworks: eg. PDL vs. Temporal Logic, PDL vs. STIT, STIT vs. ATL, etc.

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Theorem $\Box\varphi \leftrightarrow \varphi$ is provable in combinations of Epistemic Logics and PDL with certain "cross axioms" ($\Box[a]\varphi \leftrightarrow [a]\Box\varphi$) (and full substitution).

R. Schmidt and D. Tishkovsky. *On combinations of propositional dynamic logic and doxastic modal logics*. JOLLI, 2008.

Merging logics of rational agency

- Reasoning about information change (knowledge and time/actions)
- Knowledge, beliefs and certainty
- "Epistemizing" logics of action and ability: *knowing how to achieve* φ vs. *knowing that you can achieve* φ
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Conclusions

Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

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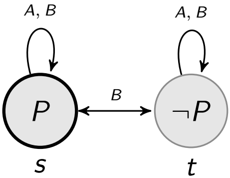
Is this procedure correct? Yes, if

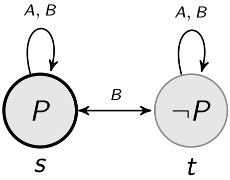
1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*

Example



P means "The talk is at 2PM".

Example



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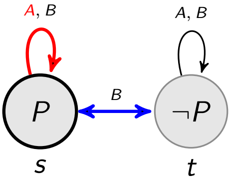

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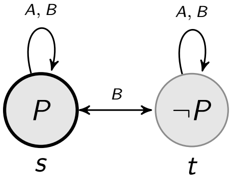

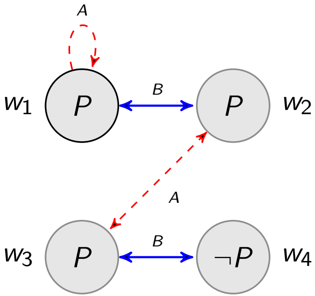

Two Methodologies

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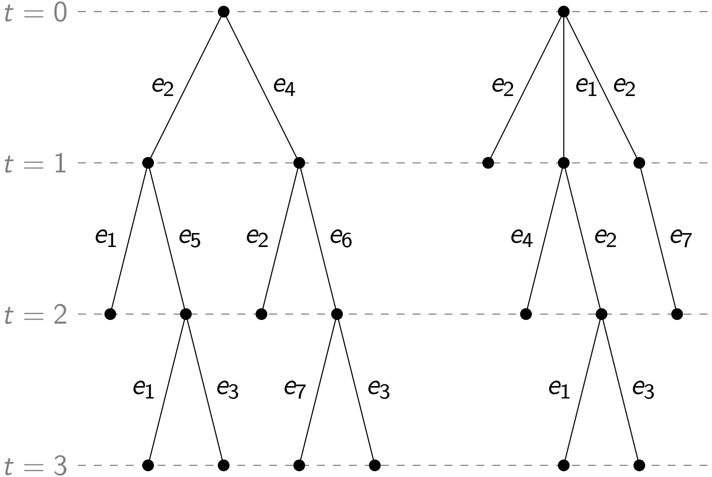
Dynamic Epistemic Logic

Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The 'Playground'



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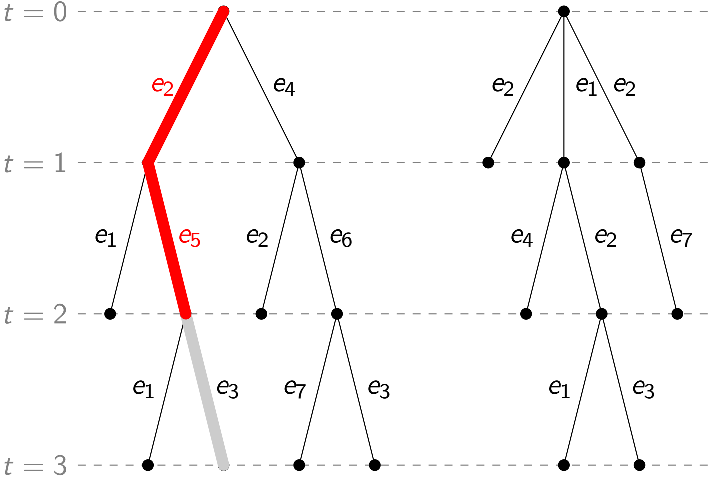

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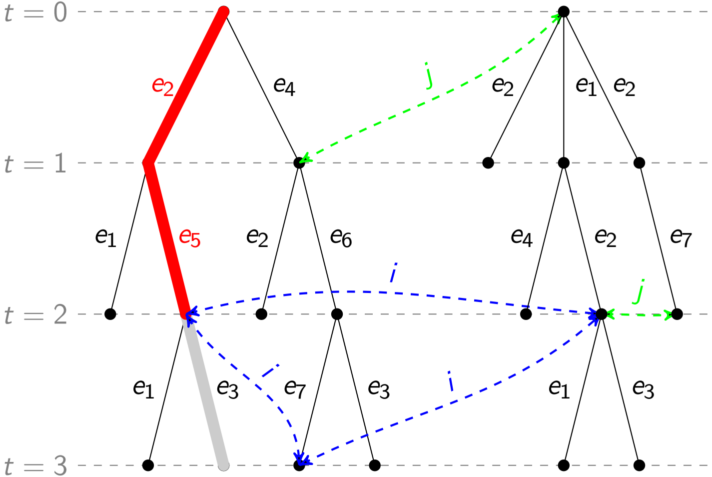

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathsf{At} \rightarrow 2^{\mathsf{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

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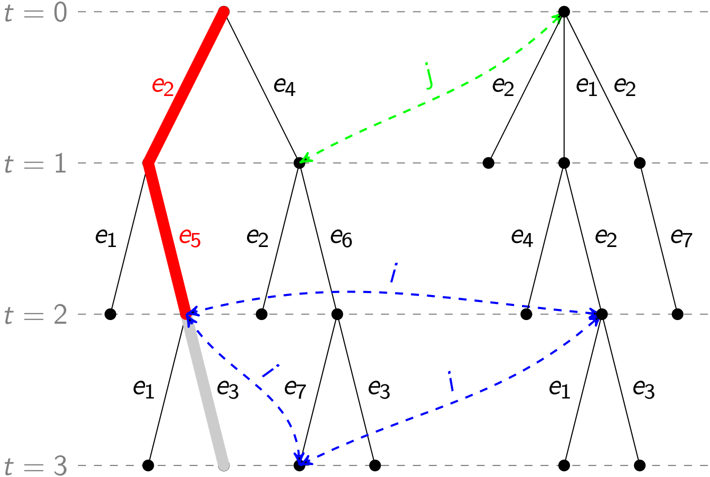
t = 0
t = 1
t = 2
t = 3
e2
e4
j
e2
e1
e2
e1
e5
e2
e6
e4
e2
e7
i
e1
e3
e7
e3
i
e1
e3

Returning to the Example

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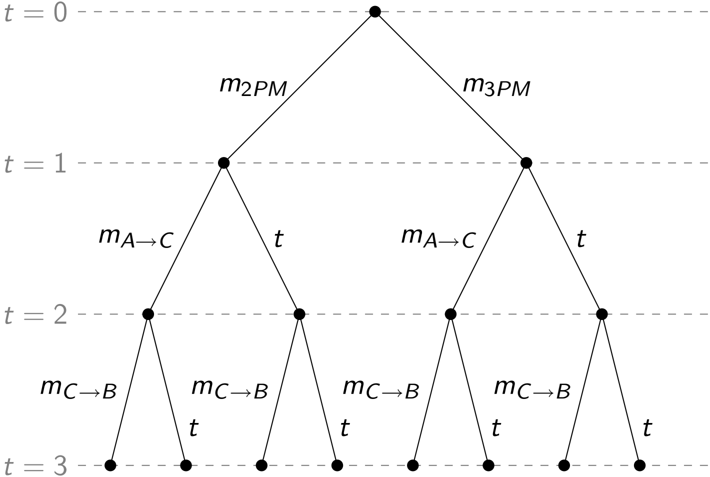
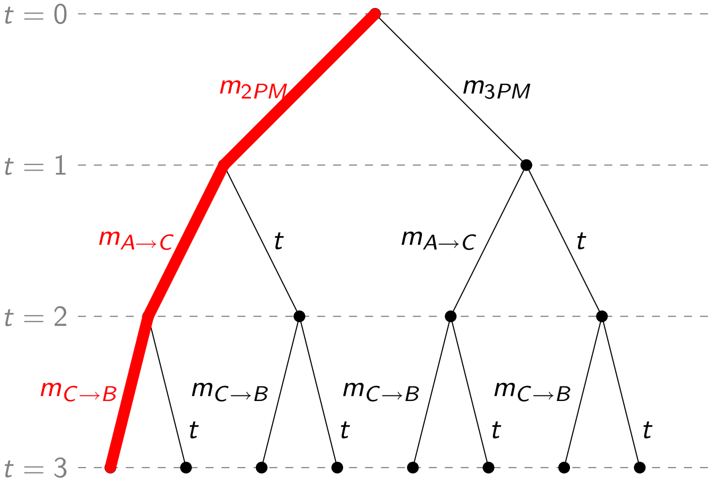
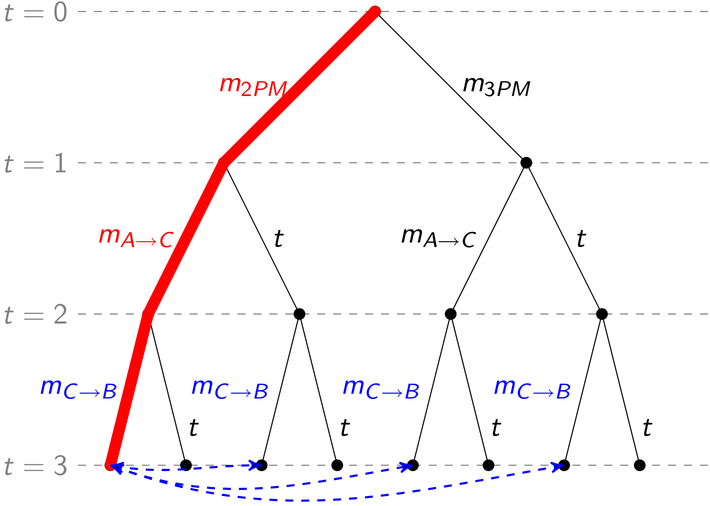
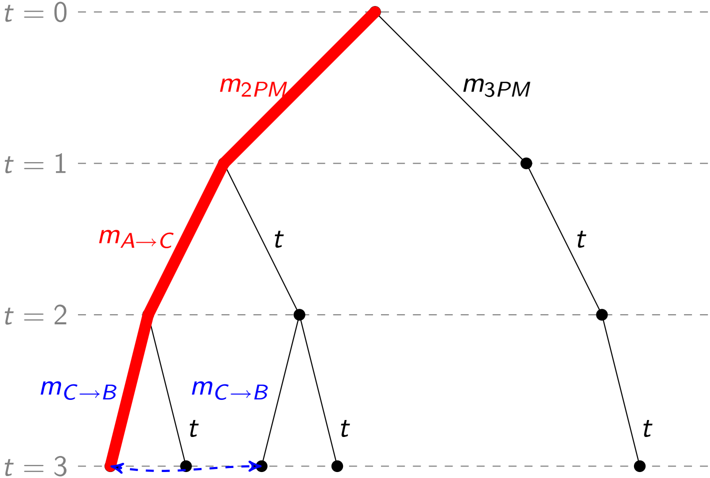

t = 0
t = 1
t = 2
t = 3
m2PM
m3PM
mA→C
t
mA→C
t
mC→B
t
mC→B
t
mC→B
t
mC→B
t



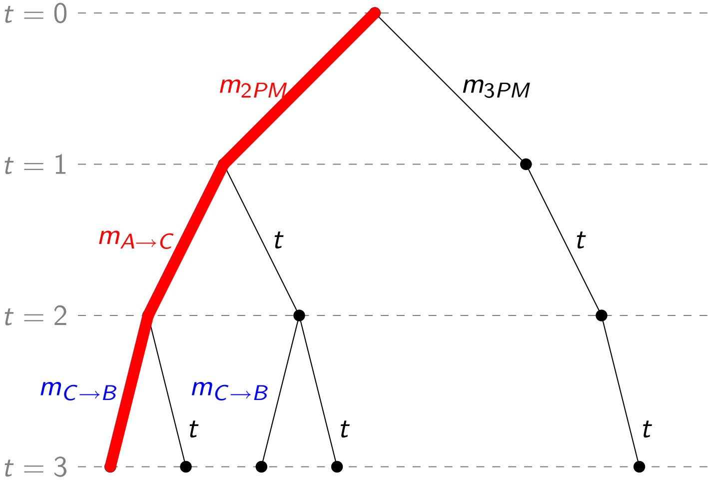
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$



Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



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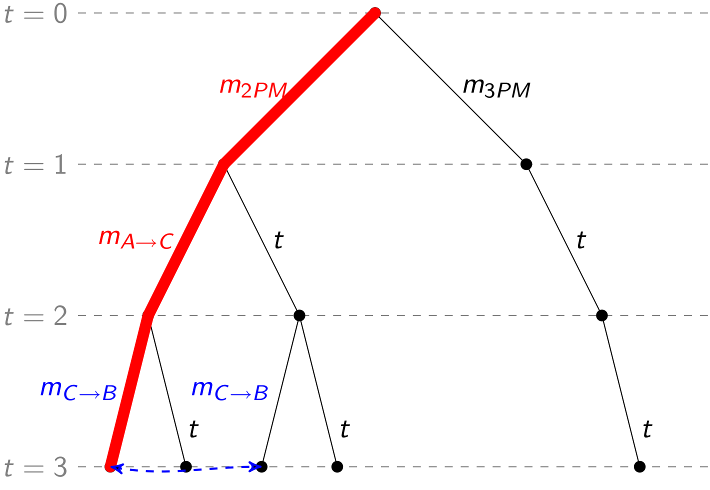

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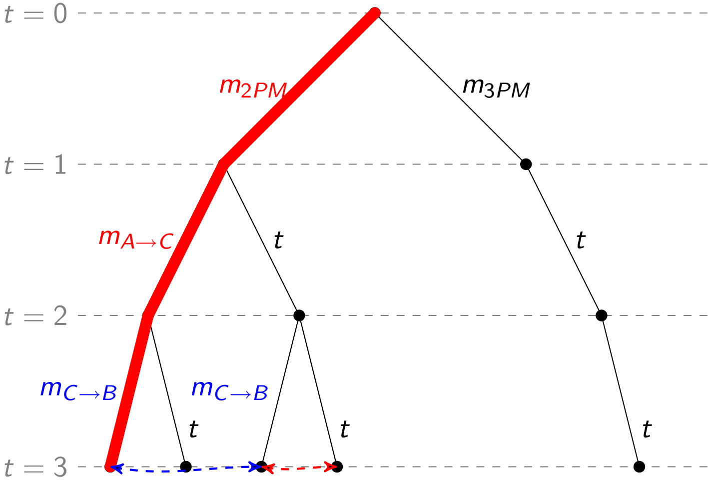

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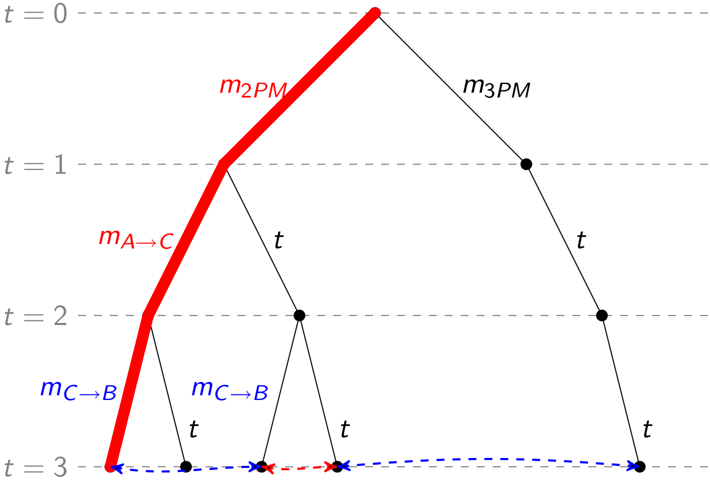

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3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

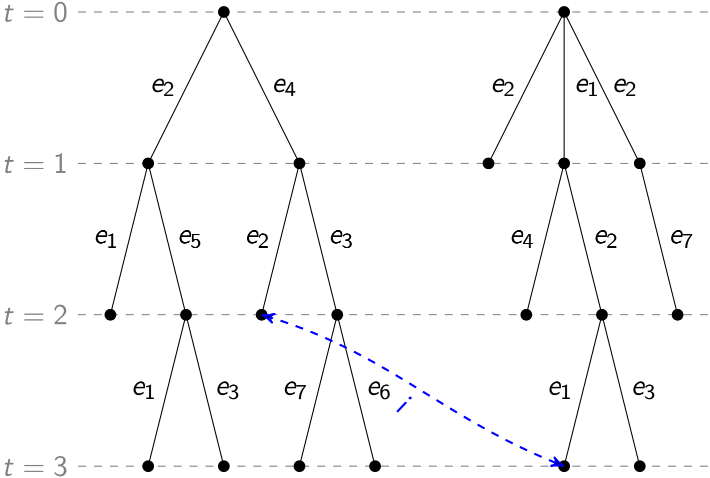
Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.

- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.

- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\text{len}(H) = \text{len}(H')$.

Perfect Recall



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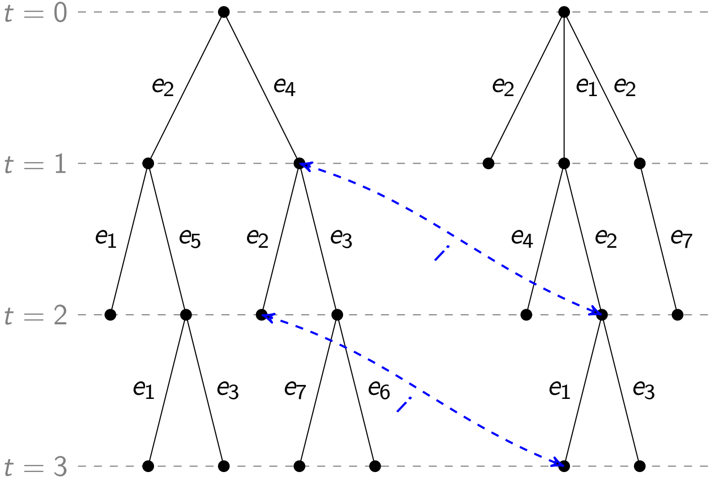

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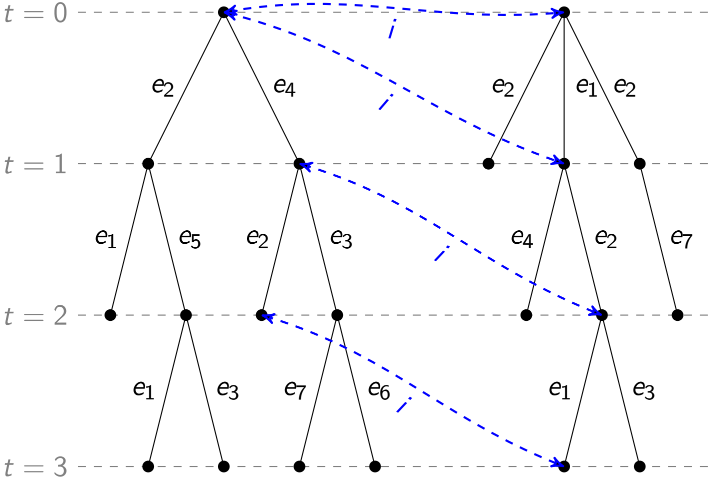

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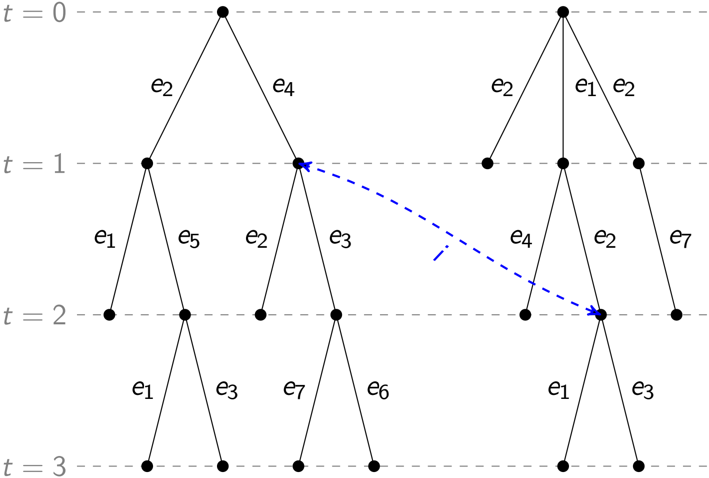

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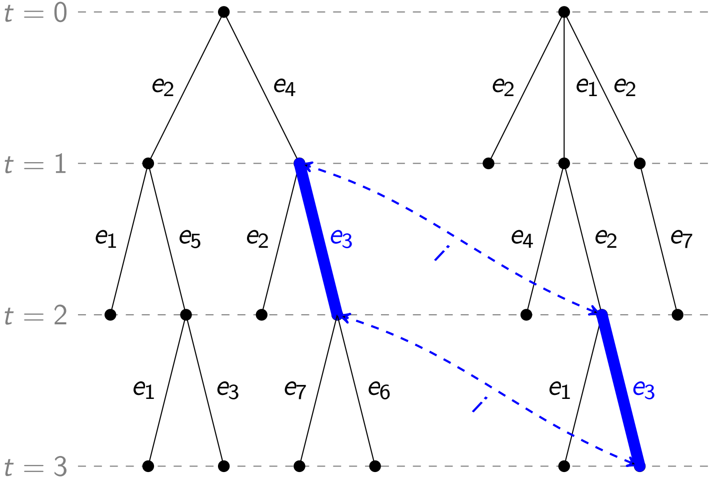

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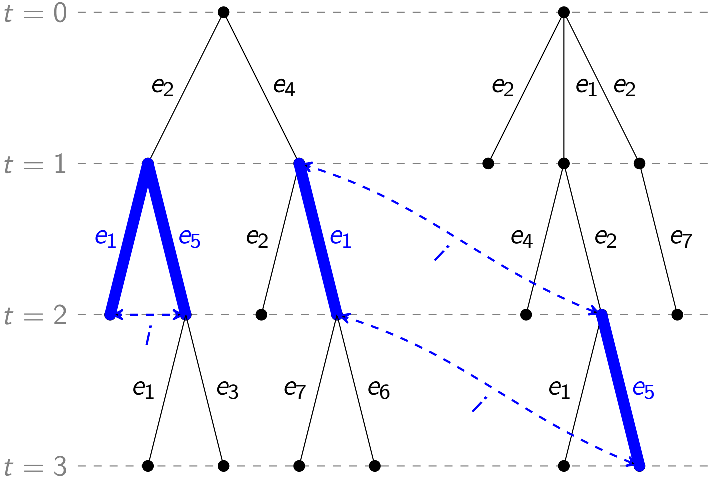

Ideal Agents

Assume there are two agents

Theorem

The logic of ideal *agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).*

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. van Benthem and EP. *The Tree of Knowledge in Action*. Proceedings of AiML, 2006.

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Dynamic Epistemic Logic

Returning to the Example: DEL

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$$(\mathcal{M} \otimes E_1) \otimes E_2$$

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The initial model (Ann and Bob are ignorant about P_{2PM}).

Private announcement to Ann about the talk.

Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

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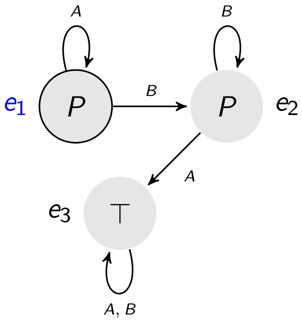

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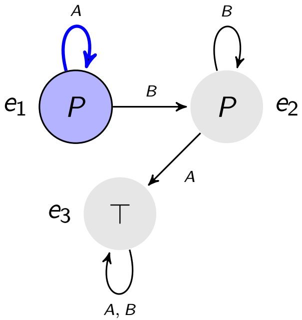

Ann knows which event took place.

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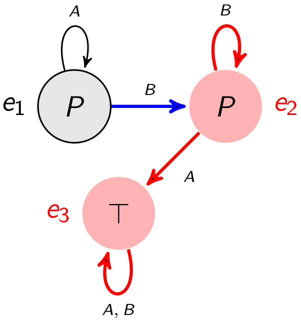

Bob thinks a different event took place.

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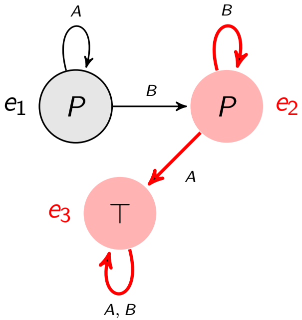

That is, Bob learns the time of the talk, but Ann learns nothing.

Product Update

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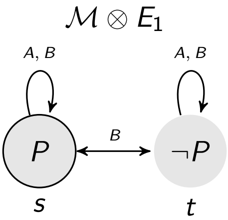
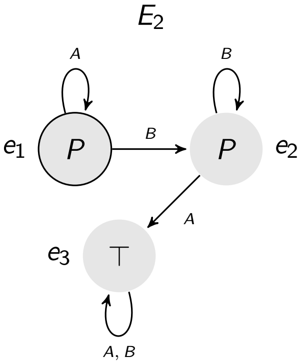
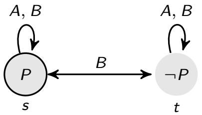
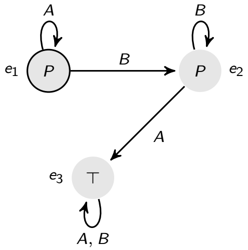
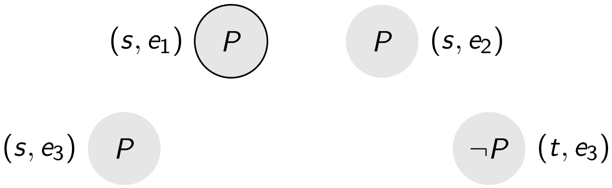




Product Update







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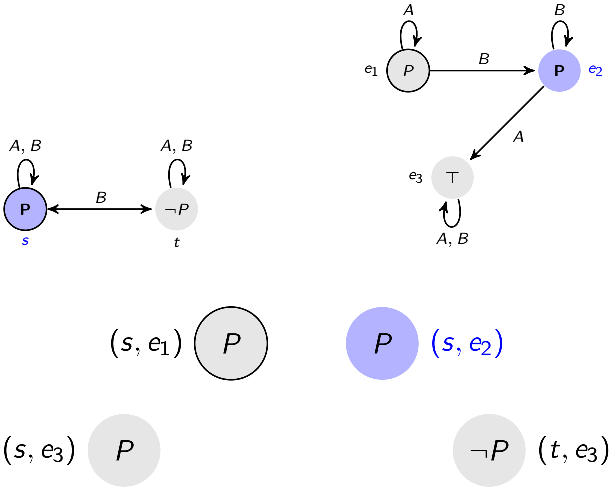

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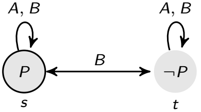
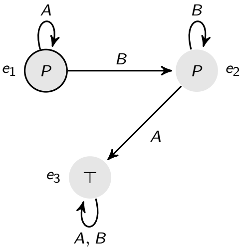
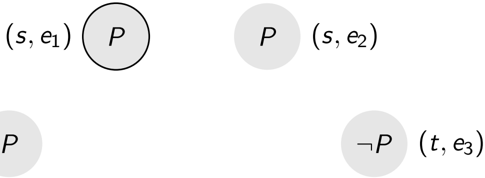




$(s, e_1) \models \neg K_B K_A K_B P$



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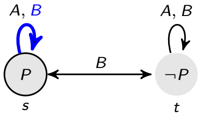

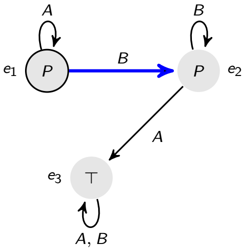

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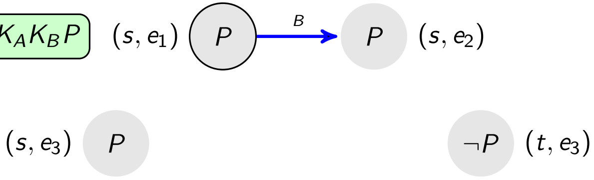

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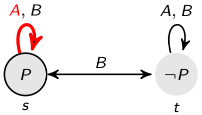

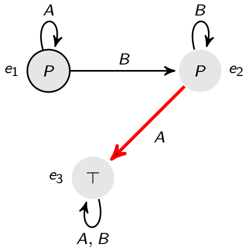

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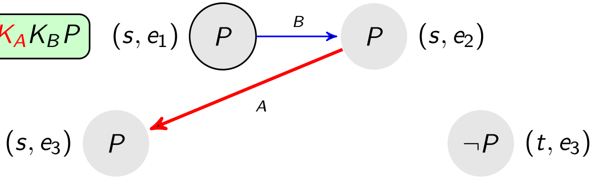

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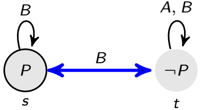

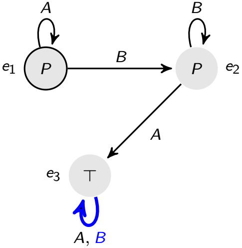

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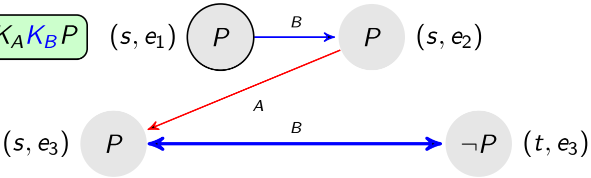

Product Update Details

Let $\mathbb{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathbb{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \to \wp(A)$.

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$\mathcal{M}, w \models [\mathrm{A}, a]\varphi$ iff $\mathcal{M}, w \models Pre(a)$ implies $\mathcal{M} \otimes A, (w, a) \models \varphi$.

Literarture

A. Baltag and L. Moss. *Logics for Epistemic Programs*. 2004.

W. van der Hoek, H. van Ditmarsch and B. Kooi. *Dynamic Episetmic Logic*. 2007.

Some Questions

- How do we relate the ETL-style analysis with the DEL-style analysis?
- In the DEL setting, what are the underlying assumptions about the reasoning abilities of the agents?
- Can we axiomatize interesting subclasses of ETL frames?

J. van Benthem, J. Gerbrandy, T. Hoshi, EP. *Merging Frameworks for Interaction*. JPL, 2009.

Skip Details

DEL *and* ETL

Observation: By repeatedly updating an epistemic model with event models, the machinery of DEL creates ETL models.

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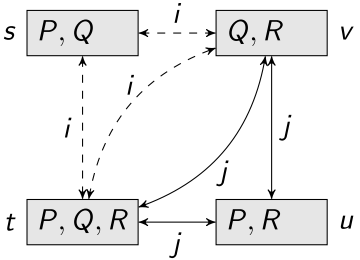
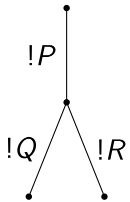
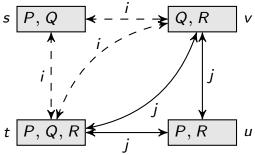
Let M be an epistemic model, and P a DEL protocol (tree of event models). The ETL model generated by M and P, forest(M, P), represents all possible evolutions of the system obtained by updating M with sequences from P.

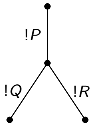
Example: Initial Model and Protocol





Example





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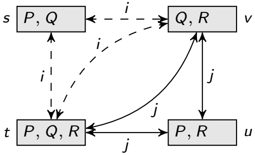
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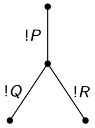
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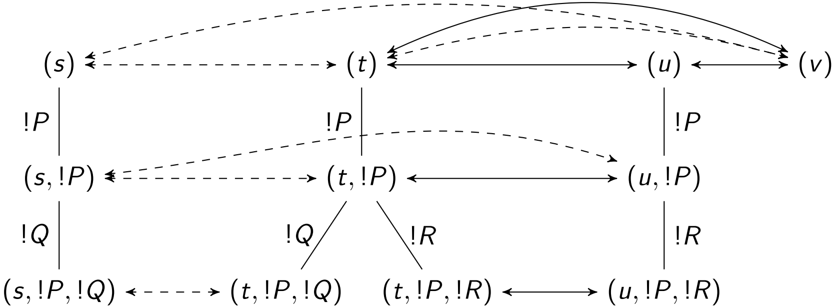
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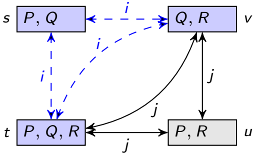
Example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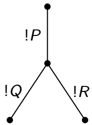
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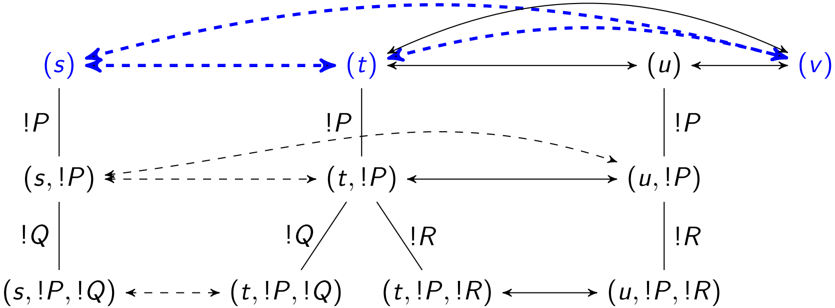
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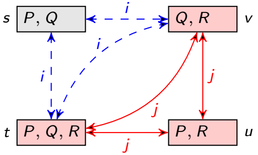
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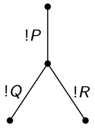
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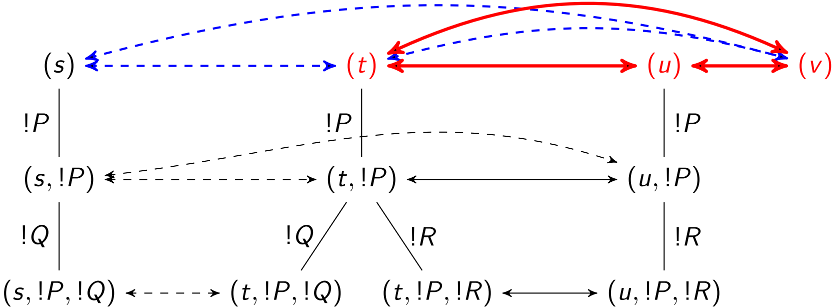
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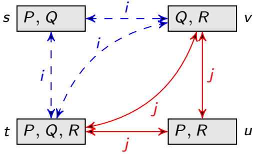
Example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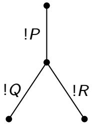
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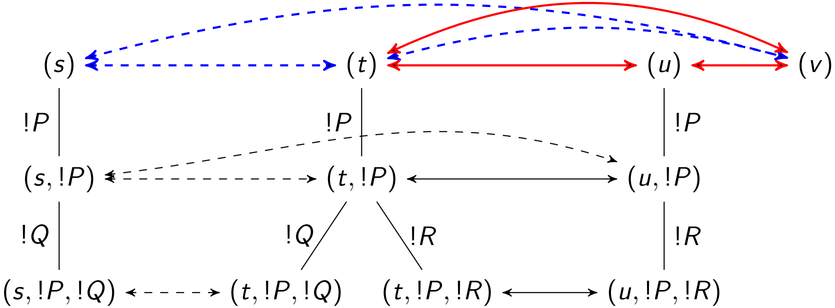
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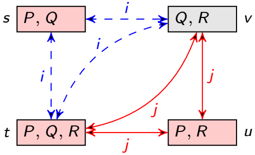
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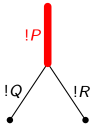
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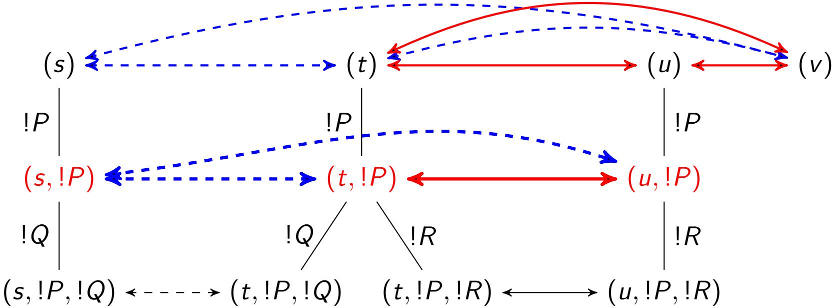
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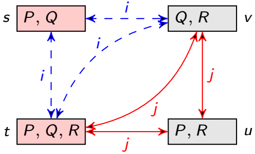
Example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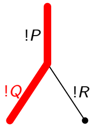
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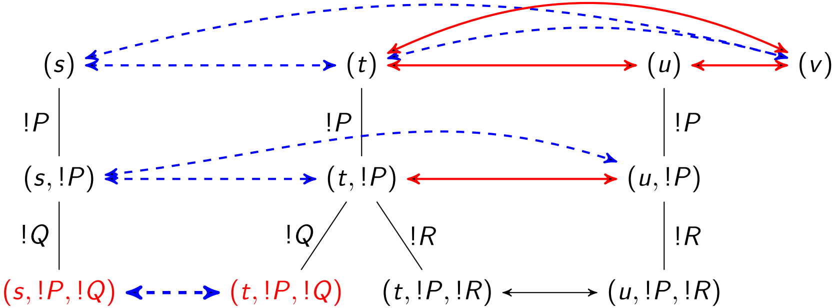
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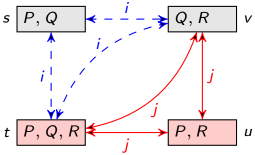
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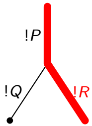
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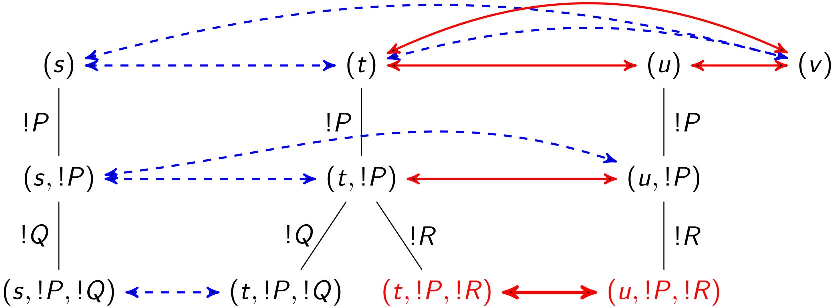
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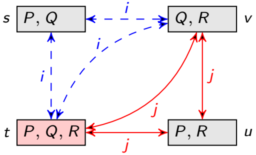
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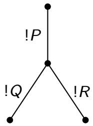
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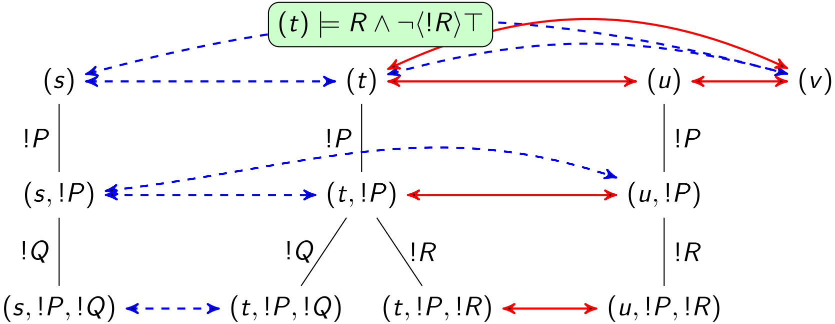
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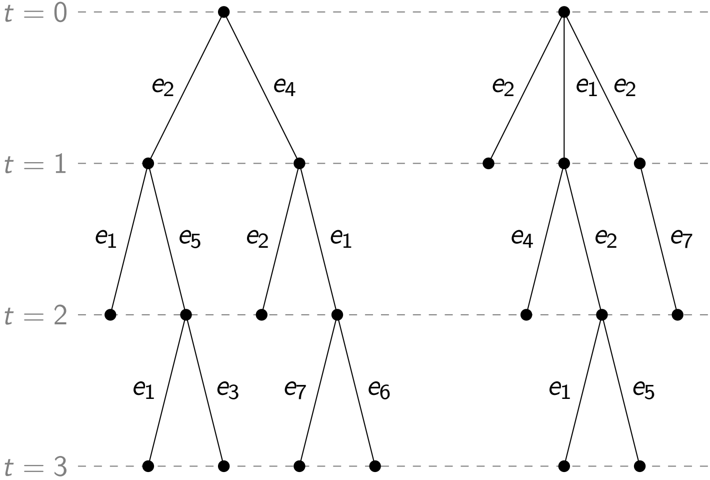
Representation Result

Given a set of DEL protocols $\mathbf{X}$, let $\mathbb{F}(\mathbf{X})$ be the class of ETL frames generated by protocols from $\mathbf{X}$.

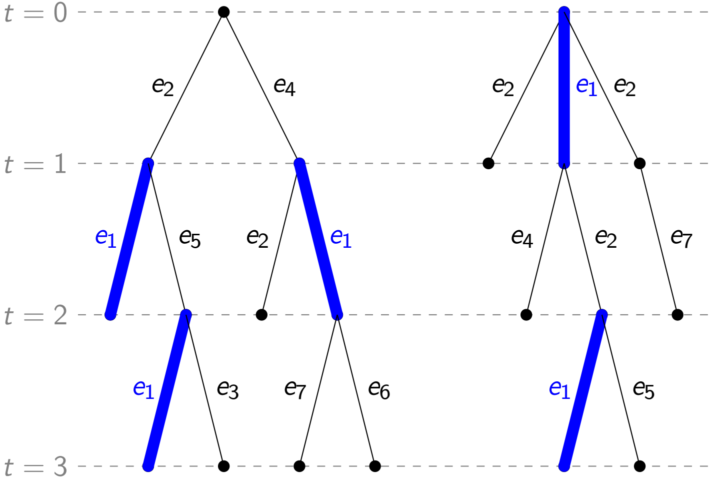
Theorem (Main Representation Theorem)

Let Σ be a finite set of events and suppose $\mathbf{X}_{DEL}^{uni}$ is the class of uniform DEL protocols (with a finiteness condition). A model is in $\mathbb{F}(\mathbf{X}_{DEL}^{uni})$ iff it satisfies propositional stability, synchronicity, perfect recall, local no miracles, and local bisimulation invariance.

Bisimulation Invariance + Finiteness Condition



Bisimulation Invariance + Finiteness Condition



Recall that if $\mathbf{X}$ is a set of DEL protocols, we define $\mathbb{F}(\mathbf{X}) = \{\mathbb{F}(\mathcal{M}, \mathsf{P}) \mid \mathcal{M} \text{ an epistemic model and } \mathsf{P} \in \mathbf{X}\}$. This construction suggests the following natural questions:

- Which DEL protocols generate interesting ETL models?
- Which modal languages are most suitable to describe these models?
- Can we axiomatize interesting classes DEL-generated ETL models?

J. van Benthem, J. Gerbrandy, T. Hoshi, EP. *Merging Frameworks for Interaction*. JPL, 2009.

Announcement + Protocol Information

1. $A \to \langle A \rangle \top$ vs. $\langle A \rangle \top \to A$

Announcement + Protocol Information

1. $A \rightarrow \langle A \rangle \top$ vs. $\langle A \rangle \top \rightarrow A$
2. $\langle A \rangle K_i P \leftrightarrow A \wedge K_i \langle A \rangle P$

Announcement + Protocol Information

1. $A \rightarrow \langle A \rangle \top$ vs. $\langle A \rangle \top \rightarrow A$

2. $\langle A \rangle K_i P \leftrightarrow A \wedge K_i \langle A \rangle P$

3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i(A \rightarrow \langle A \rangle P)$

Announcement + Protocol Information

1. $A \rightarrow \langle A \rangle \top$ vs. $\langle A \rangle \top \rightarrow A$
2. $\langle A \rangle K_i P \leftrightarrow A \wedge K_i \langle A \rangle P$
3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (A \rightarrow \langle A \rangle P)$
4. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (\langle A \rangle \top \rightarrow \langle A \rangle P)$

Announcement + Protocol Information

1. $A \rightarrow \langle A \rangle \top$ vs. $\langle A \rangle \top \rightarrow A$
2. $\langle A \rangle K_i P \leftrightarrow A \wedge K_i \langle A \rangle P$
3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (A \rightarrow \langle A \rangle P)$
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Theorems Sound and complete axiomatizations of various generated ETL models.

Conclusions

Merging logics of rational agency

- Reasoning about information change (knowledge and time/actions)
  - Knowledge, beliefs and certainty
  - "Epistemizing" logics of action and ability: *knowing how to achieve* φ vs. *knowing that you can achieve* φ
- Entangling knowledge and preferences
- Planning/intentions (BDI)

Logics of Knowledge and Preference

$K(\varphi \succeq \psi)$: "Ann knows that φ is at least as good as ψ"

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J. van Eijck. *Yet more modal logics of preference change and belief revision*. manuscript, 2009.

F. Liu. *Changing for the Better: Preference Dynamics and Agent Diversity*. PhD thesis, ILLC, 2008.

$$A(\psi \rightarrow \langle \succeq \rangle \varphi) \quad \textit{vs.} \quad K(\psi \rightarrow \langle \succeq \rangle \varphi)$$

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Should preferences be restricted to information sets?

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Should preferences be restricted to information sets?

$\mathcal{M}, w \models \langle \succeq \cap \sim \rangle \varphi$ iff there is a v with $w \sim v$ and $w \preceq v$ such that $\mathcal{M}, v \models \varphi$

$K(\psi \rightarrow \langle \succeq \cap \sim \rangle \varphi)$

Defining Beliefs from Preferences

- Starting with the work of Savage (based on Ramsey and de Finetti), there is a tradition in game theory and decision theory to *define* beliefs and utilities in terms of the agent's preferences

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- Typically the results come in the form of a representation theorem:

  If the agents preferences satisfy such-and-such properties, then there is a set of conditional probability functions and a (state independent) utility function such that the agent can be assumed to act as an expected utility maximizer.

Thus logical properties of beliefs can be derived from properties of preferences.

S. Morris. *The Logic of Belief and Belief Change: A Decision Theoretic Approach*. Journal of Economic Theory (1996).

The Framework

Let Ω be a set of states.

An **act** is a function $x : \Omega \rightarrow \mathbb{R}$. Let $\Re^{\Omega}$ be the set of all acts.

x_w for $w \in \Omega$ means that if the true state is w, then the agent receives prize x.

We write $x \succeq_w y$ the agent prefers x over y *provided the true state is w*

Belief Operators

A **belief operator** is a function $B : 2^\Omega \rightarrow 2^\Omega$

For $E \subseteq \Omega$, $w \in B(E)$ means the agent believes E at state w

B is normal if

- $B(\Omega) = \Omega$
- $B(E \cap F) = B(E) \cap B(F)$

Possibility function: $P : \Omega \rightarrow 2^\Omega$: set of states the agent considers possible at w

Defining Beliefs from Preferences

For $E \subseteq \Omega$ and two acts x and y, let (x_E, y_{-E}) denote the new act that is x on E and y on $-E$.

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B **reflects** $\{\succeq_w\}_{w\in\Omega}$ provided for each $E \subseteq \Omega$

$$B(E) = \{w \mid (x_E, y_{-E}) \sim_w (x_E, z_{-E}) \quad \text{for all } x, y, z \in \Re^\Omega\}$$

Theorem If the preference relations are complete and transitive, then the derived belief operator is normal.

S. Morris. *The Logic of Belief and Belief Change: A Decision Theoretic Approach*. Journal of Economic Theory.

▸ Conclusions

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Some Literature

Stemming from Bratman's planning theory of intention a number of *BDI logics*:

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Some common features

- Underlying temporal model
- Belief, Desire, Intention, Plans, Actions are defined with corresponding operators in a language

J.-J. Meyer and F. Veltman. *Intelligent Agents and Common Sense Reasoning*. Handbook of Modal Logic, 2007.

Bratman's Planning Theory of Intention

M. Bratman. *Intentions, Plans and Practical Reason*. Harvard University Press (1987).

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An agent commits to a (partial) plan that is

1. means-end coherent,
2. consistent with the agent's current beliefs and
3. *stable* (i.e., plans *normally* resist reconsideration) *"an agent's habits and dispositions concerning the reconsideration or nonreconsideration of a prior intention or plan determine the stability of that intention or plan". Furthermore, "The stability of [the agent's] plans will generally not be an isolated feature of those plans but will be linked to other features of [the agent's] psychology"*

Bratman's Planning Theory of Intention

Central to Bratman's theory is the idea that these partial plans direct the agent's deliberation by "constrain[ing] what options are considered relevant":

> *"plans narrow the scope of the deliberation to a limited set of options. And they help to answer a question that tends to remain unanswered in traditional decision theory, namely: where do decision problems come from?"*

A Methodological Issue

What are we formalizing? How will the logical framework be *used*?

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Two Extremes:

1. Formalizing a (philosophical) theory of rational agency: philosophers as intuition pumps generating "problems" for the logical frameworks.

2. Reasoning *about* multiagent systems. Three main applications of BDI logics: 1. a specification language for a MAS, 2. a programming language, and 3. verification language.

W. van der Hoek and M. Wooldridge. *Towards a logic of rational agency*. Logic Journal of the IGPL 11 (2), 2003.

C & L Logic of Intention

1. Intentions normally pose problems for the agent; the agent needs to determine a way to achieve them.
2. Intentions provide a “screen of admissibility” for adopting other intentions.
3. Agents “track” the success of their attempts to achieve their intentions.
4. If an agent intends to achieve p, then
   - 4.1 The agent believes p is possible
   - 4.2 The agent does not believe he will not bring abut p
   - 4.3 Under certain conditions, the agent believes he will bring about p
   - 4.4 Agents need not intend all the expected side-effects of their intentions.

C & L Logic of Intention

$$(\mathrm{PGOAL}_i p) := (\mathrm{GOAL}_i(\mathrm{LATER} p)) \wedge$$
$$(\mathrm{BEL}_i \neg p) \wedge [\mathrm{BEFORE}((\mathrm{BEL}_i p) \vee (\mathrm{BEL}_i \Box \neg p)) \neg (\mathrm{GOAL}_i(\mathrm{LATER} p))]$$

$$(\mathrm{INTEND}_i a) := (\mathrm{PGOAL}_i [\mathrm{DONE}_i(\mathrm{BEL}_i(\mathrm{HAPPENS} a))?; a])$$

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Database/Planner Picture: Planner using a database to maintain its current set of *beliefs*.

Planning vs. Database Management

1. How does an agent *generate* new intentions?
2. Given that the agent's intentions specify a *partial plan*, how and when is the plan "filled out"?
3. How does an agent choose a particular *action* (that is under its control) given its current intentions?
4. How should an agent *maintain* its current state of beliefs and intentions in the presence of new information or new intentions?
5. When should an agent *reconsider* its intentions?

Thomas Icard, EP and Yoav Shoham. *Intention and Belief Revision*. in preparation.

Our Framework

- What type of information does a planner provide? How do we represent a *plan*?
- Sources of beliefs
- Sources of dynamics: What can cause an agent's database to change?
- Changing/amending plans vs. revising/updating beliefs

Elements of a Logic of Intention Revision

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- Three views of actions: PDL (state changing), Temporal (lay out time and actions are sequences of time points), STIT (choices, or actions, constrain the future).
- Two types of beliefs: those about the state of the world and those about the future *which are governed by the agent's plans*

Intention Revision

Many of the frameworks do discuss some form of intention revision.

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W. van der Hoek, W. Jamroga and M. Wooldridge. *Towards a Theory of Intention Revision*. Synthese, 2007.

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- Intentions are derived from the agents current active plans (trees of practical reasoning rules)

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W. van der Hoek, W. Jamroga and M. Wooldridge. *Towards a Theory of Intention Revision*. Synthese, 2007.

- Two types of beliefs: strong beliefs vs. weak beliefs (beliefs that take into account the agent's intentions)
- A dynamic update operator is defined ($[\Omega]\varphi$)

Our Framework

1. *At a fixed moment*, a **choice situation** describes the current state-of-affairs (i.e., facts about the state-of-the-world), the tree of options that are available to the agent (i.e., the decision tree) and how actions change state of the world (i.e., the effect that performing an action will have on the state-of-the-world).

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2. *At a fixed moment*, a **model** describes the agent's (current) beliefs (about the current state-of-the-world and what will become true in the future including options that will become available) and the agent's (current) *instructions from the Planner* (about future choices).

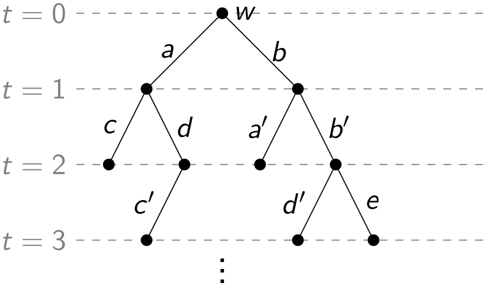
Our Framework

3. **Dynamic operators** representing each of the situations that may cause a change in beliefs and/or plans: learning a true fact, doing an action and receiving instructions from the Planner. These operators will describe how to relate models *at different moments*.

Skip Details

Choice Situations

$$\mathcal{M}_w = (W, \{R_a\}_{a \in \mathsf{Act}}, V, w)$$



Choice Situations: $\mathcal{L}_1$

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- $\mathcal{M}_w \models p$ iff $w \in V(p)$
- $\mathcal{M}_w \models \varphi \wedge \psi$ iff $\mathcal{M}_w \models \varphi$ and $\mathcal{M}_w \models \psi$
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Notation: If $\alpha = a_1a_2a_3\cdots a_n$, $\langle\alpha\rangle\varphi := \langle a_1\rangle\cdots\langle a_n\rangle\varphi$

$$N\varphi := \bigwedge_{a\in \mathrm{Act}}[a]\varphi \quad [t]\varphi := \overbrace{N...N}^{t \text{ times}}\varphi$$

$$P\varphi := \bigvee_{a\in \mathrm{Act}}\langle a\rangle\varphi \quad \langle t\rangle\varphi := \overbrace{P...P}^{t \text{ times}}\varphi$$

Adding Beliefs

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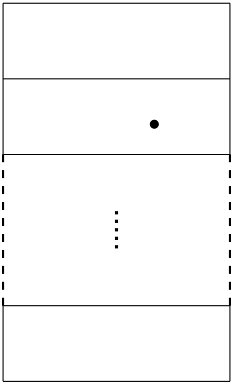
Standard picture where worlds are choice situations

$\mathcal{M}_w \preceq \mathcal{N}_v$: Choice situation $\mathcal{N}_v$ is at least as plausible as $\mathcal{M}_w$.

1. Beliefs are about available options, current and future state of affairs: $Bp \wedge B\langle a\rangle\langle b\rangle q$
2. Immediate options are *known*.
3. *In the static model*, restrict the language to only talk about *current* beliefs: $\langle a\rangle B\varphi$ is not well-formed

Belief Structures

$$\mathcal{B} = (S, \preceq, \mathcal{M}_w)$$



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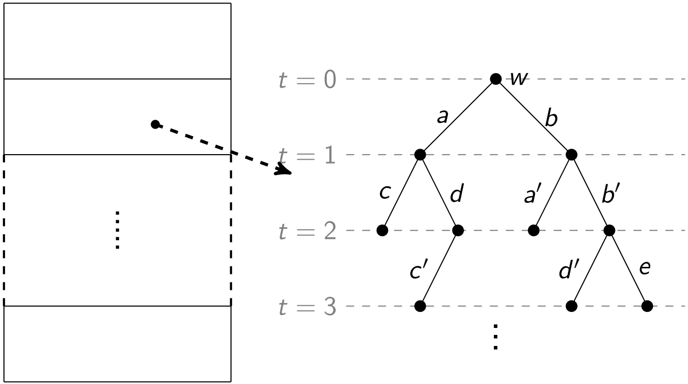

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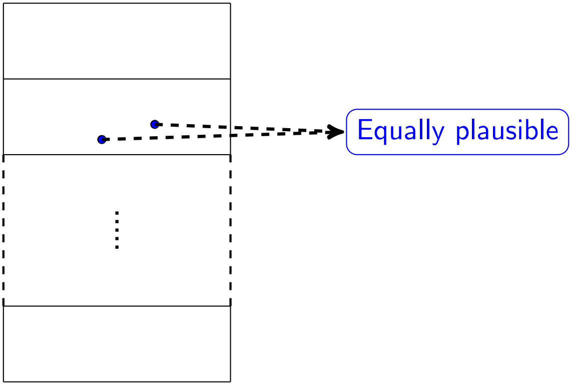

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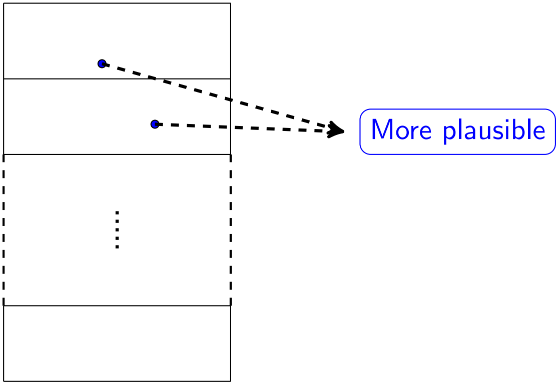

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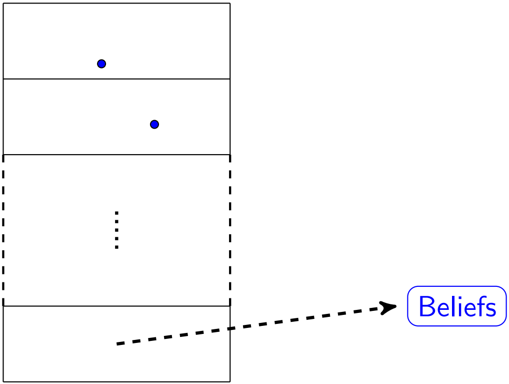

Belief Structures

Language ($\mathcal{L}_2$): $\varphi := \ \chi \mid \varphi \wedge \varphi \mid \neg\varphi \mid B(\varphi), \quad \chi \in \mathcal{L}_1$

Structures $\mathcal{B} = (S, \preceq, \mathcal{M}_w)$ is a *belief structure* if:

(i) S a set of choice situations

(ii) $\preceq$ is a plausibility ordering (reflexive, transitive, well-founded)

(iii) $\mathcal{M}_w \in S$.

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(iv) If wR_ax for some x in $\mathcal{M}$, then for all $\mathcal{N}_v \in S$ s.t. $\mathcal{M}_w \preceq \mathcal{N}_v$, there is some x' for which vR_ax' in $\mathcal{N}$.

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Belief Structures

$\mathcal{B} \Vdash \chi$, iff $\mathcal{M}_w \vDash \chi$.

$\mathcal{B} \Vdash \varphi \wedge \psi$, iff $\mathcal{B} \Vdash \varphi$, and $\mathcal{B} \Vdash \psi$.

$\mathcal{B} \Vdash \neg\varphi$, iff $\mathcal{B} \nVdash \varphi$.

$\mathcal{B} \Vdash B(\varphi)$, iff for all $\mathcal{N}_v \in Min_{\preceq}(S)$, $\mathcal{B}, \mathcal{N}_v \Vdash \varphi$.

Completeness

1. Standard proof works for the class of choice situations
2. The class of belief structures is also easily axiomatized ($\Box\varphi$ means φ is true an all worlds at least as plausible as the current world):
   - **KD45** for B
   - $\langle a\rangle\top \rightarrow \Box(\langle a\rangle\top)$
   - $\Diamond(\langle a\rangle\top) \rightarrow \langle a\rangle\top$

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5. The Planner may provide a more complicated structure (subplan structure, goals, etc.)

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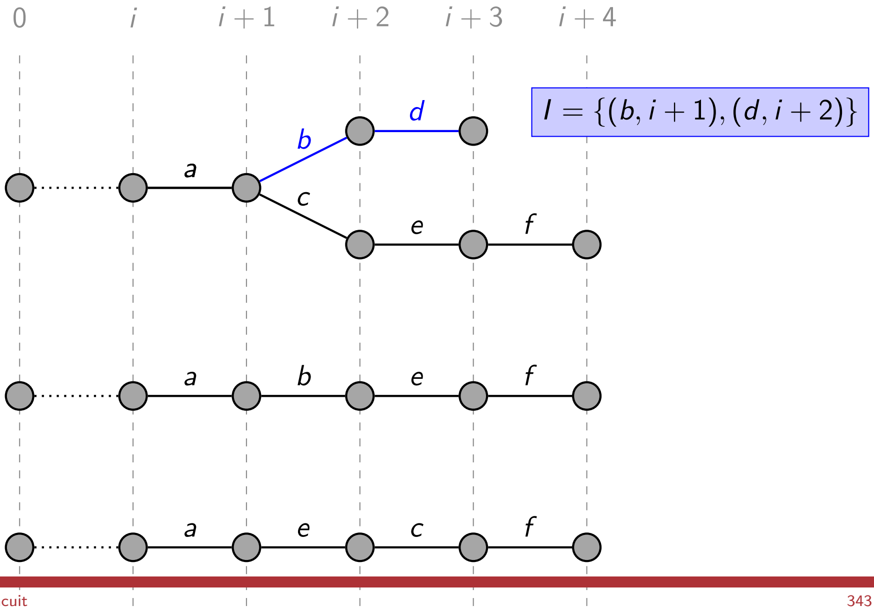
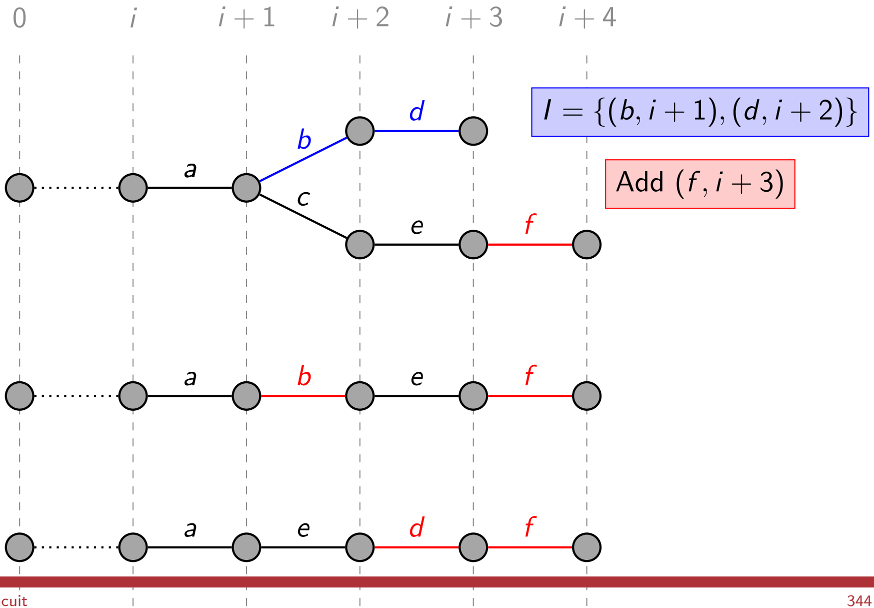
3. The Planner can amend the agent's current set of instructions.

We assume that only doing an action moves time forward. However, all three types of events may change the agent's beliefs and current instructions.


0
i
i + 1
i + 2
i + 3
i + 4
I = {(b, i + 1), (d, i + 2)}
a
b
c
d
e
f
a
b
e
f
a
e
c
f


0
i
$i+1$
$i+2$
$i+3$
$i+4$
$I = \{(b, i+1), (d, i+2)\}$
Add $(f, i+3)$
a
b
c
d
e
f
a
b
e
f
a
e
d
f

Selection Function

Say a set of *beliefs* $\mathcal{B}$ and a set of *instructions* I is **coherent** if the agent doesn't believe the instructions are impossible.
A **selection function** γ maps a set of beliefs $\mathcal{B}$ and instructions to a set of instructions: $\gamma(\mathcal{B}, I) = I'$

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3. additional principles.....

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AGM-style principles and representation theorem; Modal-style completeness (with dynamic operators get considerably more technical: *reduction axioms* are not available).

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Moving to complex plans (with choice, concatenation and test):

1. The notion of Belief-Plan consistency must be updated
2. Define intentions *semantically*: the agent "intends a, t just in case it is a *necessary component* of the current plan".
3. Many agents
4. .....

Conclusions

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- (epistemic) foundations of game theory
  Logic *and* Game Theory, not Logic in place of Game Theory.
- Social Software: Verify properties of social procedures
  - *Refine existing social procedures or suggest new ones*

R. Parikh. *Social Software*. *Synthese* **132** (2002).

Conclusions

- Many types of informational attitudes: "hard" knowledge, belief, belief about the future state of affairs, "intention" based beliefs, revisable beliefs, safe beliefs.

- Where does the "protocol" come from? What do the agents know about the protocol?

Logics of Rational Agency

- What's going on in the area:
  www.loriweb.org

- Special Issue of Synthese: Knowledge, Rationality and Interaction. *Logic and Intelligent Interaction*, Volume 169, Number 2 / July, 2009
  (eds. T. Agotnes, J. van Benthem and EP)

- New subarea of Stanford Encyclopedia of Philosophy on logic and rational agency
  (eds. J. van Benthem, EP, and O. Roy)

Calls for....

- **Papers**: LOFT 2010. University of Toulouse, July 21 - 23. Deadline: March 15, 2010.

- **Ph.D. position**: TiLPS, Tilburg University, "A formal analysis of social procedures". Deadline: **October 15** (to start in February).

Thank You!